MODULE : 02

12/05/22
Thursday

GEAR DRIVES : [SPUR GEAR & HELICAL GEAR]

Mechanical drive :-

It is a mechanism which is Intended to transmit mechanical power over certain distance usually Involving a change in speed and torque.

Gears :- Gear are also called as toothed wheels which are use to transmitt power and motion from 1 shafts to another shaft by means of Successive engagement of teeth.

* Gear drives as positive drive smooth and velocity ratio remain constant. for example :- spur, helical, Bevel, worm gear.

Classification of gears.

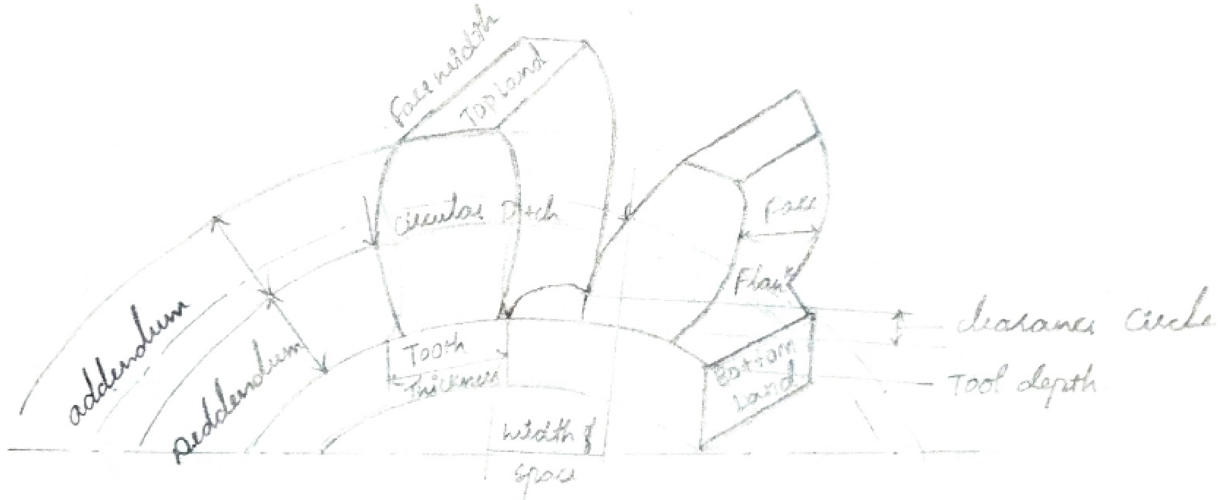
1. Based on relative positions of axis.
   * parallel gear (ex = spur and helical gear).
   * Intersecting gear (ex :- Bevel gear)
   * Non parallel and non Intersecting gear (ex :- worm gear).
2. According to type of gearing :-
   * external
   * Internal.
3. According on position of teeth on gear surface.
   * straight teeth.
   * Inclined teeth.
   * curve teeth.

4. According to pheripheral velocity of gear.

* Low velocity
* medium velocity
* high velocity

Spur Gear :- Spur gear are the gears whose axis are parallel and whose are parallel to centre line of gears. Spur gear finds its applications from small watches to gear boxes it is fitted in Aeroplane.

Spur Gear terminology :-



1. Pitch Circle :- An Imaginary circle passing through pitch point having its centre at the axis of a gear.

2. pitch point :- Common point of contact between 2 pitches circle.

3. Pitch circle diameter (PCd) :- The Diameter of pitch circle. The size of gear usually specified by PCd.

4. Circular pitch (PC) :- It is the distance measure along the circumference pitch circle from a point on 1 tooth to a corresponding point of an adjacent to

$$\boxed{P_c = \pi \times m \cdot (\sigma) \quad \pi = d/z}$$

where d = diameter
z = number of teeths.

5. Diametral pitch (Pd) :- ratio of numbers of teeth to the PCD

$$\left[Pd = \frac{z}{d} \right]$$

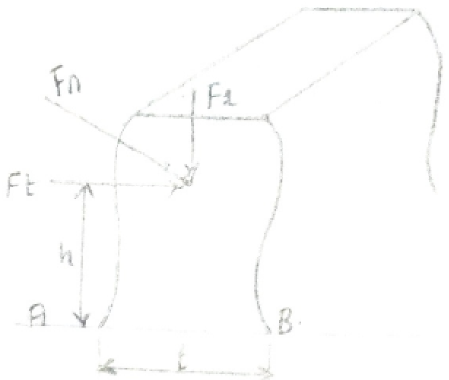
6. Module :- It is defined as ratio of pcd. to the Z.

7.

13. Pressure angle (α) angle of obliquity (ϕ)

It is the angle between common normal to gear teeth at the point of contact and the common tangent of the pitch point

Beam strength of a S teeth (or) Lewis Equation



Lewis Equation gives the beam strength of a gear tooth the load carrying ability a gear tooth is determined by Lewis Equation

Consider the tooth has a Cantilever beam subjected to gear load (Fn) as shown in figure. It is resolved into 2 Component namely tangential Component [Ft] acting tangential to the Pitch circle and radial Component [Fr] acting ⊥ to [Ft]

The tangential Component Induces bending stress which tends to Break the tooth, radial Component Induces Compressive stress of Smaller magnitude hence Fr is neglected and Ft is Considered as Basis for design calculation.

It is observed that the Cross section of tooth various from free End to fixed end. Therefore A parabola is Constructed within the tooth profile as shown by the dotted line. The advantage of parabolic outline is that its a beam of uniform strength for this beam stress at any point across the Cross section in uniform.

The weakest section of gear tooth is at section AB.

At Section AB, bending moment Equation.

$$\frac{M}{I} = \frac{\sigma_b}{y}$$

$$\sigma_b = \frac{M \times y}{I}$$

moment, $M = F_t \times h$

$$y = t/2.$$

$$I = \frac{bh^3}{12} = \frac{bt^3}{12}$$

$$\sigma_b = \frac{F_t \times h \times \left(\frac{t}{2}\right)}{\frac{bt^3}{12}}$$

$$\sigma_b = \frac{6 F_t \times h}{bt^2}$$

$$\therefore \boxed{F_t = \frac{\sigma_b \cdot bt^2}{6h}} \longrightarrow (1)$$

In the above expression t and h are variables depending upon the size of the tooth i.e circular pitch and its profile.

Consider $t = x \times P_c$ and $h = K \times P_c$.

where, x and K are Constants

$$F_t = \frac{\sigma_b \; b \; x . P_c^{\cancel{2}}}{6 \times K \; \cancel{P_c}}$$

$$F_t = \frac{\sigma_b . b \, x^2 \, P_c}{6 \times K}$$

Let. $\frac{x^2}{6K} = y.$

$$\boxed{F_t = \sigma_b \times b \times y \times P_c}$$

The Quantity y is known as lewis form factor / tooth FF.
the value of y for various tooth system is given by.

$$\boxed{y = 0.124 - \frac{0.684}{z}}$$ → For $14\frac{1}{2}$ Involute of System.

$$\boxed{y = 0.154 - \frac{0.912}{z}}$$ → For 20° Involute System

$$\boxed{y = 0.175 - \frac{0.95}{z}}$$ → For 20° stubtooth Involute System.

The Permissible stress in the lewis Equation depends upon material pitch line velocity and loading Conditions.

According to Barth's formula,
Permissible stress given by $\boxed{\sigma_b = \sigma_0 \times C_v}$

where, σ_0 = alowable static stress.
C_v = velocity factor.

∴ Final form of lewis Equation,

$$\boxed{F_t = \sigma_0 \times C_v \times b \times y \times P_c}$$

σ_b is replaced by σ_0.

System of Gear tooth :-

There are 3 Commonly used gear teeth System namely

1. 14.5 Involute System.
2. 20° full depth Involute System.

3. 20° stub tooth Involute System

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Dynamic tooth load [Fd] :-

when 2 gears mesh together and transmit power gear teeths experience much higher load near the tangential tooth load, this is due to the following reasons.

1. Error in tooth space.
2. Inaccuracy in tooth profile.
3. Non-uniform distribution of load.
4. Deflection of tooth and shaft under load.

As a result of Inaccuracies there will dynamic load due to shock and Impact. The dynamic load will be > the steady load $\boxed{Fd = Ft + Fi}$ It consists of tangential load [Ft] and Incremental load [Fi] Caused by Irregularities. The dynamic load in the sum of $\boxed{Fd + Fi}$

i.e $Fd = [Fd + Fi]$

$$\boxed{Fd = Ft + \frac{K_3 V [Cb + Ft]}{K_3 V + \sqrt{Cb + Ft}}} \rightarrow \text{[eq 12.12 pg 207]}.$$

Wear load Capacity [Fw] :-

when paint of gear marks together the teeth slide Over each other from begining of engagement to the end as results the teeth where out, Since the pinion rotates faster than the gear and has less number of teeth. The teeth on pinion moves at faster rate

Buckingham's presented Equation for wear resistance.

$$\boxed{F_W = d_1 b Q K} \longrightarrow (12.15(a)\ Pg\ 208)$$

Design Procedure for Spur Gear :-

STEP : 1 :- Identify the weaker member.

To decide the weaker member among the tooth the following table has to be formulated.

Particular	σ_d	y	$\sigma_d \times y$	remarks.
1 is always correspond to this PINION	σ_{d_1}	y_1	$(\sigma_{d_1} \times y_1)$	
GEAR. 2 is always correspond to this	σ_{d_2}	y_2	$(\sigma_{d_2} \times y_2)$	

The members which smaller value of $\sigma_d \times y$ is weaker member, design is based for the weaker member.

STEP : 2 :- Design based on strength of weaker members.

★ Tangential tooth load $= F_t = \dfrac{1000 P C_s}{V}$ (12.7 (d) Pg 205)

where, P = power in kW.

C_s = service factor (Table 12.8)

★ tangential tooth load on lewis Equation.

$F_t = \sigma_o \times C_v \times bY \longrightarrow$ (12.6(a) to 12.6(c) Pg 205)

C_v = velocity factor.

B = Is equal to the face mill.

* By Equating above 2 Equation using trial and Error method find module 'M' adopted standard value of 'M' from (table 12.2 Pg 229).

STEP 3 :-

Calculate all Important geomatric parameter of tooth Profile i.e addendum, Deddendum, tooth thickness, total depth, clearance, Outer diameter etc (from table 12.3 Pg 229).

STEP 4 :- check for dynamic and wear loads :-

* dynamic loads experianced by gear on pinion is given by $\boxed{F_d = F_t + F_i}$ and $\boxed{F_d = F_t + K_3}$

$$\boxed{F_d = F_t + \frac{K_3 V (Cb + F_t)}{K_3 V + \sqrt{Cb + F_t}}} \longrightarrow \text{(eq 12.12 Pg 207)}.$$

where, $K_3 = 20.67$

V = velocity

b = face width

C = Co-eff that depends on material pressure angle and Error in an action.

* check for wear load capacity according to Buckingham's Equation,

$$\boxed{\text{i.e. } F_W = d_1 b Q K}$$

where, Q = ratio factor

$$Q = \frac{2d_2}{d_1 + d_2} \quad (or) \quad \frac{2Z}{Z_1 + Z_2}$$

K = load stress factor.

STEP : 5 :- For safe design $\boxed{F_w > F_d}$

Problems :-

1. A spur gear pinion 100mm diameter as a torque of 200Nm applied to it the spur gear meshes with a 250mm diameter gear the pressure angle is 20°. Determine the tangential force, radial force and torque on gear.

Given data

Pinion	Gear
$d_1 = 100mm$	$d_2 = 250mm$
T_1 (or) $m_{t1} = 200Nm$	$T_2 = ?$
$\phi = 20°$	
$F_t = ?$	
$F_r = ?$	

1. For pinion (T_1)

WKT,

Torque. $T = F_{t1} \times r$.

$$(200 \times 10^{-3}) = F_{t1} \times \left(\frac{d_1}{2}\right)$$

$$F_{t1} = \frac{200 \times 10^{-3}}{\left(\frac{100}{2}\right)}$$

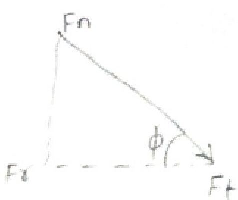
$$\boxed{F_{t1} = 4000N}$$

2. To find radial for (F_r) = ?

$$\tan\phi = \frac{F_{r1}}{F_{t1}}$$

$$\tan(20) = \frac{F_{r1}}{4000}$$

$$\boxed{F_{r1} = 1455.88N}$$



3. For gear (m_{t1}) = ?

m_{t2} = Torque × radius.

$$m_{t2} = F_{t2} \times r_2$$

$$= 4000 \times \left(\frac{250}{2}\right)$$

$$m_{t2} = 500 \times 10^3 \, N\text{-}mm$$

$$\boxed{m_{t2} = 500 \, N\text{-}m}$$

$F_{t1} = F_{t2}$

$F_{r1} = F_{r2}$

2. Design a pair of spur gear to transmit 20KW power from a shaft rotating at 1000rpm. to a shaft rotating at 310rpm. Assume Number of teeth on pinion is 31. and 20° full depth Involute tooth form the material for pinion is C45 steel untreated and for gear 0.2% cast steel for carbon untreated.

Given data :- (Type 1 problem) (If they ask No of teeth be along type 1 prob)

$P_1 = 20KW$. $\phi = 20°FDI$.

N_1 = always greater

N_2 = lesser value

If they not give ϕ assume $\phi = 20°$ Involute

Spur gear	gear
$N_1 = 1000rpm$	$N_2 = 310rpm$
$Z_1 = 31$	$Z_2 = ?$
Material :-	material.
C45 steel untreated	0.2% Carbon untreated. — cast steel

WKT, velocity / speed ratio $i = \frac{N_1}{N_2} = \frac{Z_2}{Z_1} = \frac{d_2}{d_1}$

No of teeth of gear Z_2,

$$\frac{N_1}{N_2} = \frac{Z_2}{Z_1} \Rightarrow \frac{1000}{310} = \frac{Z_2}{31} \Rightarrow \boxed{Z_2 = 100\text{ teeth}}$$

[From table 12.7 Pg 234].

For pinion [for allowable static stress $\sigma_{d1} = 233.4 N/mm^2$]

For gear [for allowable static stress $\sigma_{d2} = 138.3 N/mm^2$]

Lewis form factor (y) = ?

$$y_1 = \left(0.154 - \frac{0.912}{z_1}\right) \longrightarrow \text{(eq 12.5 (c) Pg 204)}$$

$$y_1 = \left(0.154 - \frac{0.912}{31}\right)$$

$$\boxed{y_1 = 0.12}$$ for pinion

$$y_2 = \left(0.154 - \frac{0.912}{z_2}\right) \longrightarrow \text{(eq 12.5 (c) Pg 204)}$$

$$y_2 = \left(0.154 - \frac{0.912}{100}\right)$$

$$\boxed{y_2 = 0.144}$$ for gear.

Solutions,

STEP : 1 : Identify the weaker member.

Particular	σ_d	y	$\sigma_d \times y$	Remarks.
PINION	233.4	0.124	28.94	
GEAR	138.3	0.144	19.91 ✓	weaker.

Since $\sigma_d \times y$ of gear is lesser than pinion. The design is based on gear. (gear is the weaker members).

STEP : 2 :- Design Based on strength of gear.

Lewis Equation for beam strength of a tooth is given by.

$$m = \left[\frac{2\, m_t}{\sigma_d \cdot C_v \times K Y Z}\right]^{\frac{1}{3}} \longrightarrow \text{(from fig eq}^n \text{ 12.5b Pg 204)}$$

To find torque $(m_t) = ?$

$$M_{t2} = \frac{9.55 \times 10^6 \times P \times C_s}{n_2}$$

$$= \frac{9.55 \times 10^6 \times 20 \times 1.50}{310}$$

$$\boxed{M_{t2} = 924.19 \times 10^3 \, N\text{-}mm}$$

(from table 12.8 Pg 235)

Assume always medium shock and 8-10 h/day.

Service factor $\boxed{C_s = 1.50}$

Substitute P always in kW

To find $K = ?$

$$K = \frac{b}{m} = \frac{10m}{m}$$

$$\boxed{K = 10}$$

$b = 9.5m$ to $12.5m$,

(from eq 12.5 (f) Pg 205)

To find $Y = ?$

$$\pi y_2 = \pi \times (0.144) = \boxed{Y = 0.452}$$

Substitute all the above values in lewis Equation

$$M = \left[\frac{2M_t}{\sigma_d \times C_v \times K \times Y \times Z}\right]^{\frac{1}{3}}$$

$$= \left[\frac{2 \times 924.19 \times 10^3}{138.3 \times C_v \times 10 \times 0.452 \times 100}\right]$$

$$\boxed{m^3 C_v = 29.57}$$

Assume C_v = (The velocity factor for the spur gear which ranges from 0.325 to 0.6.

(assume $C_v = 0.5$)

$$m^3 = \frac{29.57}{0.5}$$

$$\boxed{M = 3.89}$$

To check for module $(m) = ?$

WKT, $m = \frac{d}{z}$ (eq 12.1 (d) Pg 203)

$4 = \frac{d}{100}$

$\boxed{d = 400mm}$

(From table 12.2 pg 229)

Assume preffered choice 1 Standard module

$\boxed{m = 4mm}$

To find velocity $(v) = ?$

$$v = \frac{\pi d_2 n_2}{60{,}000} = \frac{\pi \times 400 \times 310}{60{,}000} \Rightarrow \boxed{v = 6.49 m/s}$$

To find velocity factor $(C_v) = ?$

Barth's formula $C_v = \frac{3.05}{3.05 + v}$ → (from Eq 12.6 (a) pg 205)

$$C_v = \frac{3.05}{3.05 + 6.49}$$

$$\boxed{C_v = 0.319}$$

$$(m^3 C_v)_{new} = (4^3 \times 0.319) \Rightarrow \boxed{(m^3 \times C_v)_{new} = 20.416}$$

Since $(m^3 C_v)_{new}$ is $> (m^3 C_v)_{required}$. The design is not safe

Assume $m = 5mm$ (from table 12.2 pg 229).

WKT,

$$m_2 = \frac{d}{z} = 5 = \frac{d}{100} \Rightarrow \boxed{d = 500mm}$$

$$v_2 = \frac{\pi d_2 n_2}{60{,}000} = \frac{\pi \times 500 \times 310}{60{,}000} \Rightarrow \boxed{v_2 = 8.11 m/s}$$

$C_v = \frac{4.58}{4.58 + v}$ → (from Eq 12.6 (b) pg 205).

$$= \frac{4.58}{4.58 + 8.11} \Rightarrow \boxed{C_v = 0.36}$$

$$(m^3 C_v)_{new} = (5^3 \times 0.36)$$

$$\boxed{(m^3 C_v)_{new} = 45.11}$$

Since $(m^3 C_v)_{new}$ is $>$ $(m^3 C_v)$ required. The design is safe $\therefore$ standard $m = 5mm$,

STEP: 3 :- Dimension,

Calculate all the dimensions (from table 12.3 pg 229).

Addendum (h_a)	$= m$	$= 5mm$	
Dedendum (h_f)	$= 1.25m$	$= 1.25 \times 5$	$= 6.25$
Tooth thickness (t)	$= 1.5708m$	$= 1.5708 \times 5$	$= 7.854$
Tooth space	$= 1.5708m$	$= 1.5708 \times 5$	$= 7.854$
Working depth	$= 2m$	$= 2 \times 5$	$= 10$
whole depth	$= 2.25m$	$= 2.25m$	$= 11.25$
clearance	$= 0.25m$	$= 0.25m$	$= 1.25$

STEP 4 :- check for dynamic and wear load,

$$F_d = F_t + \frac{K_3 v (C_b + F_t)}{K_3 v \sqrt{(C_b + F_t}} \Rightarrow \text{(Eq 12.12 pg 207)}.$$

$$d_2 = m \times Z_2 \Rightarrow 5 \times 100 \Rightarrow \boxed{d_2 = 500mm}$$

WKT, To find $\cdot F_{t_2} = ?$

$$m_{t_2} = F_{t_2} \times \text{radius}$$

$$m_{t_2} = F_{t_2} \times \frac{d_2}{2}$$

$$924.19\times10^3 = Ft_2 \times \left(\frac{500}{2}\right)$$

$$\boxed{Ft_2 = 3696.72\,N}$$

To find $K = ?$ $\boxed{K = 20.67}$

To find $C = ?$

Dynamic factor (c) $\Rightarrow$ (from Pg no 236 Table 12.12).
For module of 5mm

(Assuming Class II error = 0.0277mm).

To find error refer 12.13 Table

If they not mention anything always choose accurate (I) carefully cut gear.

For 20° Full depth Involute system.
materials is (steel and steel).

$\boxed{E = 0.0277}$ $\Rightarrow$ (from table 12.13 Pg 237).

For error = 0.02 = 228.9

0.0277 C

$$\boxed{\text{Dynamic factor } C = 317.02}$$

To find $b = ?$

$b = 10m = 10\times5 \Rightarrow \boxed{b = 50\,m}$

$$F_d = F_t + \frac{K_3 V (Cb + F_t)}{K_3 V \sqrt{Cb + F_t}}$$

$$= 3696.76 + \frac{20.67\times8.11\,(317.02\times50 + 3696.76)}{20.67\times8.11 + \sqrt{317.02\times50 + 3696.76}}$$

$$\boxed{F_d = 14.35\times10^3\,N}$$

STEP : 5 :- wear load.

$d_1 = m \times Z_1 \Rightarrow 5 \times 31 \Rightarrow \boxed{d_1 = 155mm}$

$$Q = \frac{2Z_2}{(Z_2 + Z_1)} \Rightarrow \frac{2 \times 100}{(100 + 31)} \Rightarrow \boxed{Q = 1.51}$$

For safe design,

$F_W \geq F_d$

$d_1 b Q K \geq F_d$

$(155 \times 50 \times 1.51 \times K) \geq 14.35 \times 10^3$

$\boxed{K \geq 1.226}$

For steel and steel, $\phi = 20^\circ$ and $K \geq 1.226$.

(from table 12.16 pg 239).

For pinion BHN = 350
For gear BHN = 250.

3. A Cast steel spur gear pinion having 21 teeths rotating at 1500rpm is required to transmit 9kW to a high grade Cast iron (CI). to run at 500rpm the teeths are 14.5° Involute Design a spur gear completely.

Given data :-

Pinion	Gear	
		$P = 9kW$
$Z_1 = 21$	$Z_2 = ?$	$\phi = 14.5^\circ$
$N_1 = 1500rpm$	$N_2 = 500rpm$	

To find $Z_2 = ?$

$$\frac{N_1}{N_2} = \frac{Z_2}{Z_1} \Rightarrow \frac{1500}{500} = \frac{Z_2}{21} \rightarrow \boxed{Z_2 = 63}$$

(from table 12.7 pg 234)

For pinion ($\sigma d_1 = 138.3 MPa$)

For gear ($\sigma d_2 = 78.5 MPa$).

Lewis form factor $(y) = ?$

$$y_1 = \left(0.124 - \frac{0.684}{Z_1}\right) \rightarrow \text{from eq 12.5(c) pg 204.}$$

$$y_1 = \left(0.124 - \frac{0.684}{21}\right)$$

$\boxed{y_1 = 0.091}$ for pinion.

$$y_2 = \left(0.124 - \frac{0.684}{Z_2}\right) \rightarrow \text{from eq 12.5(c) pg 204.}$$

$$y_2 = \left(0.124 - \frac{0.684}{63}\right)$$

$\boxed{y_2 = 0.113}$ for gear.

STEP: 1: Identify the weaker members,

Particular	σd	y	$\sigma d \times y$	remarks.
PINION	138.3	0.091	12.58	
GEAR.	78.5	0.113	8.87	weakers.

Since $\sigma d \times y$ of gear is lesser than pinion. The design is based on gear.

STEP 2: Design based on strength of gear.

To find torque $(m_t) = ?$

$$M_{t_2} = \frac{9.55 \times 10^6 \times P \times C_s}{n_2}$$

(from table 12.8 Pg 235)

$$\boxed{C_s = 1.50}$$

$$= \frac{9.55 \times 10^6 \times 9 \times 1.50}{500}$$

$$\boxed{M_{t_2} = 257.85 \times 10^3 \text{ N-mm}}$$

To find $y = ?$

$$\pi y_2 = \pi \times (0.113)$$

$$\boxed{Y = 0.354}$$

$$K = \frac{b}{m} = \frac{10m}{m} \Rightarrow \boxed{K = 10}$$

$$m = \left[\frac{2 \times M_t}{\sigma_d \times C_v \times K \times Y \times Z}\right]^{\frac{1}{3}} \rightarrow \text{(eq}^n \text{ 12.5b pg 204)}$$

$$m = \left[\frac{2 \times 257.85 \times 10^3}{78.5 \times C_v \times 10 \times 0.354 \times 63}\right]^{\frac{1}{3}}$$

$$\boxed{m^3 C_v = 29.45}$$

Assume from $C_v = 0.325$ to 0.6.

(Assume $C_v = 0.5$) $\quad m^3 = \frac{29.45}{0.5} \Rightarrow \boxed{m = 3.89\text{mm}}$

To check for the module $m = ?$

(From table 12.2 Pg 229)

$$\boxed{m = 4\text{mm}}$$

$$m = \frac{d}{z} \quad \text{(eq 12.1 (d) Pg 203)}$$

$$4 = \frac{d}{63}$$

$$\boxed{d = 252}$$

To find velocity $v = ?$

$$v = \frac{\pi d_2 n_2}{60{,}000} = \frac{\pi \times 252 \times 500}{60{,}000} \Rightarrow \boxed{v = 6.59\ m/s}$$

To find velocity factor $(C_v) = ?$

Barth's formula $C_v = \dfrac{3.05}{3.05 + V}$ → (from eq 12.6(a) pg 205)

$$C_v = \frac{3.05}{3.05 + 6.59}$$

$$\boxed{C_v = 0.319}$$

$$(m^3 C_v)_{new} = (4^3 \times 0.319) \Rightarrow \boxed{(m^3 \times C_v)_{new} = 20.224}$$

Since $(m^3 C_v)_n$ is $>$ $(m^3 C_v)_R$ the design is not safe.

Assume $m = 5mm$, (from table 12.2 pg 229)

WKT,

$$d_2 = m \times Z_2 = 5 \times 63 \Rightarrow \boxed{d_2 = 315}$$

$$V_2 = \frac{\pi d_2 n_2}{60{,}000} = \frac{\pi \times 315 \times 500}{60{,}000} \Rightarrow \boxed{V_2 = 8.24\ m/s}$$

$$C_v = \frac{4.58}{4.58 + V} = \frac{4.58}{4.58 + 8.24} \Rightarrow \boxed{C_v = 0.357}$$

$$(m^3 C_v)_{new} = (5^3 \times 0.357)$$

$$\boxed{(m^3 C_v)_{new} = 44.625}$$

Since $(m^3 C_v)_{new}$ is $>$ $(m^3 C_v)_R$ The design is safe. and the standard $m = 5mm$.

STEP: 3:- Dimensions. (from table 12.3 pg 229)

Addendum $(h_a) = m = 5mm$.

deddendum $(h_f) = 1.25m = 1.25 \times 5 = 6.25$.

Tooth thickness $(t) = 1.5708m = 1.5708 \times 5 = 7.854$.

Tooth space $= 1.5708m = 1.5708 \times 5 = 7.854$.

working depth $= 2m = 2 \times 5 = 10$

whole depth $= 2.25m = 2.25 \times 5 = 11.25$.

clearance $= 0.25m = 0.25 \times 5 = 1.25$.

STEP: 4:- check for dynamic.

$$F_d = F_t + \frac{K_3 V (C_b + F_t)}{K_3 V \sqrt{C_b + F_t}} \rightarrow \text{(from eq}^n \text{ 12.12 pg 207)}$$

WKT, To find $F_{t2} = ?$

$$M_{t2} = F_{t2} \times r$$

$$M_{t2} = F_{t2} \times \frac{d_2}{2}$$

$$257.85 \times 10^3 = F_{t2} \times \frac{315}{2}$$

$$\boxed{F_{t2} = 1637.14N}$$

To find $K = ?$ $\boxed{K = 20.67}$

To find $b = ?$ $b = 10m = 10 \times 5 \Rightarrow \boxed{b = 50}$

To find $C = ?$

Class II error $= 0.0277$ mm. $\longrightarrow$ (from table 12.13 Pg 237)

material (C5 and C1)

For error $= \frac{0.02}{0.0277} = \frac{151.6}{C}$

$\boxed{C = 209.96}$

$$F_d = F_t + \frac{K_3 v \sqrt{(Cb + F_t)}}{K_3 v \sqrt{Cb + F_t}}$$

$$= 1637.14 + \frac{20.67 \times 8.24\,(209.96 \times 50 + 1637.14)}{20.67 \times 8.24 \sqrt{209.96 \times 50 + 1637.14}}$$

$\boxed{F_d = 9006.3\,N}$

STEP : 5 : wear load,

$d_1 = m \times z_1 = 5 \times 21 \Rightarrow \boxed{d_1 = 105\,mm}$

$$Q = \frac{2z_2}{(z_2 + z_1)} = \frac{2 \times 63}{(63 + 21)} \Rightarrow \boxed{Q = 1.5}$$

For safe design,

$F_W \geq F_d$.

$d_1 b Q K \geq F_d$

$(105 \times 50 \times 1.5 \times K) \geq 9006.3$

$\boxed{K = 1.14}$

For steel and Cast iron $\phi = 14.5°$ and $K \geq 1.14$.

(from table 12.16 Pg 239)

$\boxed{\begin{array}{l} \text{For pinion CS BHN} = 250 \\ \text{For gear CI BHN} = 180 \end{array}}$

4. Two spur gears are to be used for a rock crusher drive and are to be of minimum size. the gears are to be designed for the following requirement power to be transmitted 18KW speed of the pinion 1200 rpm. velocity ratio 3.5 : 1. tooth Profile 20° Stub tooth Involute System. Determine module and face width for strength requirement only.

<u>Given data</u>:-

(rock crusher drive)

rock crusher means heavy gear

$P = 18KW$

$N_1 = 1200 rpm$

$i = 3.5 : 1$

$\phi = 20°$ for stub tooth Involute System.

module $(m) = ?$

$b = ?$

<u>Solution</u>:

$$i = \frac{N_1}{N_2} \Rightarrow 3.5 = \frac{1200}{N_2} \Rightarrow \boxed{N_2 = 342.85 rpm}$$

Since the number of teeth on pinion and gear are not given let us assume min No of teeth on pinion $Z_1 = 20$ inch (Ta 12.4 Pg 230

$$i = \frac{z_2}{z_1} \Rightarrow 3.5 = \frac{z_2}{20} \Rightarrow \boxed{z_2 = 70 \text{ teeth}}$$

(As the gear drive is to be of minimum / compact in size. The strength of the gear should be maximum select stronger material for both pinion and gear. (from table 12.7 pg 234). Assume chromium vanadium steel 0.45% carbon for both pinion and gear material.

[From table 12.7 pg 234]

$(\sigma d_1 = 516.8 N/mm^2) \rightarrow$ for pinion.

$(\sigma d_2 = 516.8 N/mm^2) \rightarrow$ for gear,

Always assume PSG material as same (it is not similar anything)

Lewis form factor : Y = ?

$$y_1 = \left(0.175 - \frac{0.95}{z_1}\right) = \left(0.175 - \frac{0.95}{20}\right)$$

$$\boxed{y_1 = 0.1275}$$

$$y_2 = \left(0.175 - \frac{0.95}{z_2}\right) \rightarrow (\text{eq } 12.5 (c) \text{ pg } 204) = \left(0.175 - \frac{0.95}{70}\right)$$

$$\boxed{y_2 = 0.1614}$$

STEP 1: Identify the weaker member.

Particle	σd	y	$\sigma d \times y$	remarks.
PINION	516.8	0.1275	65.892	weaker
GEAR	516.8	0.1614	83.411	

Since $\sigma d \times y$ of pinion is lesser than gear. The design is based on pinion.

STEP : 2 :- Design based on strength of gear.

$$Mt_1 = \frac{9.55 \times 10^6 \times P \times Cs}{n_1}$$

(from table 12.8 pg 235)

$\boxed{Cs = 1.50}$

$$= \frac{9.55 \times 10^6 \times 18 \times 1.50}{1200}$$

$\boxed{Mt_1 = 214.87 \times 10^3}$

$\pi y_2 = \pi \times 0.1275 \Rightarrow \boxed{Y = 0.400}$

$K = \frac{b}{m} = \frac{10\cancel{m}}{\cancel{m}} \Rightarrow \boxed{K = 10}$

$$m = \left[\frac{2\,Mt}{\pi d_1 \times Cv \times K \times Y \times Z}\right]^{1/3} \rightarrow \text{(from eq}^n \text{ 12.5b pg 204)}$$

$$m = \left[\frac{2 \times 214.87 \times 10^3}{516.8 \times Cv \times 10 \times 0.400 \times 20}\right]^{1/3}$$

$\boxed{m^3 Cv = 10.39}$

Assume $Cv = 0.325$ to 0.6.

(Assume $Cv = 0.5$) $\quad m^3 Cv = \frac{10.39}{0.5} \Rightarrow \boxed{m = 2.74}$ mm.

To check for the module m = ?

$m = \frac{d}{z} \rightarrow$ (eq 12.1 (a) pg 203)

(from table 12.2 pg 229)

$\boxed{m = 3\,mm}$

$3 = \frac{d}{20}$

$\boxed{d = 60}$

To find velocity $v = ?$

$$v = \frac{\pi d_1 n}{60{,}000} = \frac{\pi \times 60 \times 1200}{60{,}000} \Rightarrow \boxed{v = 3.76\,m/s}$$

To find velocity factor $C_v = ?$

Barth's formula $C_v = \frac{3.05}{3.05 + v} \rightarrow$ (from eq 12.6(a) pg 205)

$$C_v = \frac{3.05}{3.05 + 3.76}$$

$$\boxed{C_v = 0.447}$$

$$(m^3 C_v)_{new} = (3^3 \times 0.447) \Rightarrow \boxed{(m^3 C_v)_{new} = 12.069}$$

Since $(m^3 C_v)_{new}$ is $> (m^3 C_v)_r$ the design is safe. Std $m = 3$ mm.

STEP 3 : Dimensions :-

Addendum (h_a) = m = 3 mm

dedendum (h_f) = $1.25m = 1.25 \times 3 = 3.75$

Tooth thickness (t) = $1.5708m = 1.5708 \times 3 = 4.7115$

Tooth space = $1.5708m = 1.5708 \times 3 = 4.7115$

working depth = $2m = 2 \times 3 = 6$

whole depth = $2.25m = 2.25 \times 3 = 6.75$

clearance = $0.25m = 0.25 \times 3 = 0.75$

5. Design a pair of spur gear to transmit power of 18 KW. from a shaft running at 1000rpm. to a parallel shaft running at 250rpm. maintaining center distance of 160mm. between shaft centers. suggest suitable hardness for gear pair.

Given data :-

$P = 18\,KW$

$N_1 = 1000\,rpm$

$N_2 = 250\,rpm$

$a = 160\,mm$

WKT,

Speed ratio $i = \frac{N_1}{N_2} = \frac{Z_2}{Z_1} = \frac{d_2}{d_1}$

$$i = \frac{1000}{250} \Rightarrow \boxed{i = 4}$$

$$i = \frac{d_2}{d_1} = d_2 = 4d_1$$

Centre distance $a = \frac{d_1 + d_2}{2}$ → (from pg 203 eq 12.3 (a))

$$160 = \frac{d_1 + 4d_1}{2}$$

$$160 \times 2 = 5d_1 \Rightarrow \boxed{d_1 = 64\,mm}$$

$$d_2 = 4d_1 \Rightarrow 4 \times 64 \Rightarrow \boxed{d_2 = 256\,mm}$$

To find material = ?

commonly take always CS 0.2%

Assume material for both the pinion and gear is made of cast steel 0.20% carbon heat treated.

For pinion ($\sigma_{d1} = 193.2$ MPa)

For gear ($\sigma_{d2} = 193.2$ MPa).

Assume pressure angle

$\phi = 20°$ FDI

STEP: 1 Identify the weaker member.

Since both pinion and gear are made up of same material. pinion is the weaker member, so the design is based on pinion,

STEP 2 :- Design based on strength of gear.

$$F_t = \sigma_d C_v b Y m \rightarrow \text{(from eq 12.5 (a) pg 204)}$$

$$V = \frac{\pi d_1 n_1}{60{,}000} = \frac{\pi \times 64 \times 1000}{60{,}000} \Rightarrow \boxed{V = 3.35 \text{ m/s}}$$

$\boxed{C_s = 1.50} \rightarrow$ (from table 12.8 pg 235).

$$F_t = \frac{1000 \times P \times C_s}{V} \longrightarrow \text{(from eq}^n \text{ 12.7 (a) pg 205)}$$

$$= \frac{1000 \times 18 \times 1.50}{3.35} \Rightarrow \boxed{F_t = 8.05 \times 10^3 \text{ N}}$$

To find b = ?

$\boxed{b = 10m}$

To find lewis form factor. (Y) = ?

$$y_1 = \left(0.154 - \frac{0.912}{z_1}\right) \longrightarrow \text{(eq 12.5 (d) pg 204)}$$

$$y_1 = \left(0.154 - \frac{0.912}{\left(\frac{d_1}{m}\right)}\right)$$

$$y_1 = \left(0.154 - \frac{0.912\,m}{64}\right)$$

$$\boxed{y_1 = 0.154\,m}$$

$$C_v = \frac{3.05}{3.05+V} = \frac{3.05}{3.05+3.35} \Rightarrow \boxed{C_v = 0.476}$$

Substitute all the values in lewis equation

$$F_{t1} = \sigma_{d1} \times C_v \times b \times \pi y_1 \times m. \quad (12.5(a)\ 205)$$

$$8.05 \times 10^3 = 193.92 \times 0.47 \times 10m \times \pi \times (0.154 - 0.014m) \times m.$$

$$8.05 \times 10^3 = 2.85 \times 10^3 m^2 \,(0.154 - 0.014m)$$

$$8.05 \times 10^3 = 438.9m^2 - 39.9m^3.$$

$$39.9m^3 - 438.9m^2 + 8.05 \times 10^3 = 0$$

On solving module $(m) = -3.704,\ 7.359,\ 7.35$

Chose atleast (+ve) value of m i.e 7.359.

(from table 12.2 pg 229) standard module $\underline{\underline{m = 8mm}}$

WKT, $m = \frac{d}{z} = 8 = \frac{64}{z} \Rightarrow \boxed{Z_1 = 8 \text{ teeth}}$

$$Z_2 = \frac{d_2}{m} = \frac{256}{8} \Rightarrow \boxed{Z_2 = 32 \text{ teeth}}$$

STEP: 3:- Dimensions:-

(from table 12.3 pg 229).

Addendum (ha) = m = 8mm

dedendum (hf) = 1.25m = 1.25 × 8 = 10

Tooth thickness (t) = 1.5708m = 1.5708 × 8 = 12.56

Tooth space = 1.5708 m = 1.5708 × 8 = 12.56

working depth = 2m = 2 × 8 = 16

whole depth = 2.25m = 2.25 × 8 = 18

clearance = 0.25 m = 0.25 × 8 = 2

STEP : 4 :- Check for dynamic.

$$F_d = F_t + \frac{K_3 V (Cb + F_t)}{K_3 V \sqrt{Cb + F_t}}$$ → (from eqⁿ 12.12 pg 207)

WKT, To find 'C'

For 0.03 = 343.3.

0.0386 = C

$\boxed{C = 441.7}$

b = 10m = 10 × 8 = 80mm

$$F_d = 8.05 \times 10^3 + \frac{20.67 \times 3.35 (441.7 \times 80 + 8.05 \times 10^3)}{20.67 \times 3.35 \sqrt{441.7 \times 80 + 8.05 \times 10^3}}$$

$\boxed{F_d = 18.87 \times 10^3 N}$

STEP : 5 :- wear load.

$$Q = \frac{2Z_2}{(Z_2+Z_1)} = \frac{2\times 32}{32+8} \Rightarrow \boxed{Q = 1.6}$$

For safe design,

$$F_W \geq F_d$$

$$d_1 b Q K \geq F_d$$

$$(64 \times 80 \times 1.6 \times K) \geq 18.87 \times 10^3$$

$$\boxed{K \geq 2.305}$$

For steel and steel $\phi = 20°$ FDI and $K \geq 2.305$.

(from table 12.16 Pg 239)

For pinion steel BHN	= 450
For gear steel BHN	= 350.

6. A pair of carefully cut class II spur gear transmit 20KW. at 230 rpm of the gear and reduction ratio is 5:1 The Pinion is made of Cast steel heat treated with allowable stress of 197 MN/m² gear is made of Cast iron with allowable stress of 56 MN/m². Determine module, face width, number of teeth on pinion and gear. also suggest suitable surface hardness (BHN) for the gear pair. pitch line velocity of the pinion is not to exceed 7.5 m/s.

<u>Given data:-</u>

$P = 90KW$

$N_2 = 230rpm$

$i = 5:1$

$\sigma d_1 = 197 \, MN/m^2$

$\sigma d_2 = 56 \, MN/m^2$

$m = ?$

$b = ?$

$BHN = ?$

$V \leq 7.5m/s.$

<u>Solution:-</u>

$$i = \frac{N_1}{N_2} \Rightarrow 5 = \frac{N_1}{230} \Rightarrow \boxed{N_1 = 1150rpm}$$

$$V = \frac{\pi d_1 n_1}{60,000} \Rightarrow 7.5 = \frac{\pi \times d_1 \times 1150}{60,000} \Rightarrow \boxed{d_1 = 124.55mm}$$

$$i = \frac{d_2}{d_1} \Rightarrow 5 = \frac{d_2}{120} \Rightarrow \boxed{d_2 = 600mm}$$

*** Assume temparasily the min number of teeths on pinion i.e $\boxed{Z_1 = 20 \text{ teeths}}$ Just to identify the weak members

$$i = \frac{Z_2}{Z_1} \Rightarrow 5 = \frac{Z_2}{20} \Rightarrow \boxed{Z_2 = 100 \text{ teeths}}.$$

<u>Lewis form factor:-(Y) = ?</u>

$$y_1 = \left(0.154 - \frac{0.912}{Z_1}\right) \longrightarrow (\text{from eq}^n \text{ 12.5 (c) pg 204})$$

$$y_1 = \left(0.154 - \frac{0.912}{20}\right)$$

$$\boxed{y_1 = 0.1084} \text{ mm}$$

$$y_2 = \left(0.154 - \frac{0.912}{Z_2}\right) \longrightarrow (\text{from eq}^n \text{ 12.5 (c) pg 204})$$

$$y_2 = \left(0.154 - \frac{0.912}{100}\right)$$

$$\boxed{y_2 = 0.14488} \text{ mm}$$

STEP : 1 :- Identify the weaker member :-

Particle	σ_d	y	$\sigma_d \times y$	remarks.
PINION	197	0.1084	21.35	
GEAR	56	0.144	8.06 ✓	weaker.

STEP : 2 : Design based on strength of gear.

$$F_t = \sigma d_2 \times C_v \times b \times Y \times m \longrightarrow \text{(from eq}^n \text{ 12.5 (a) Pg 204)}$$

$$F_t = \frac{1000 P C_s}{V_2} \longrightarrow (12.7 \text{ (a) Pg 205})$$

$$= \frac{1000 \times 20 \times 1.50}{7.22}$$

$$\boxed{F_t = 4155.12 N}$$

$$V_2 = \frac{\pi d_2 n_2}{60,000}$$

$$= \frac{\pi \times 600 \times 230}{60,000}$$

$$\boxed{V_2 = 7.22 m/s}$$

$$\boxed{b = 10 m}$$

$$y_2 = 0.154 - \frac{0.912}{\left(\frac{d_2}{m}\right)}$$

$$y_2 = 0.154 - \frac{0.912 m}{600}$$

$$\boxed{y_2 = 0.154 - 1.52 \times 10^{-3} m}$$

$$Z_2 = \frac{d_2}{m}$$

don't take (0.144) y_2 value because its temporarily

$$C_v = \frac{3.05}{3.05 + 7.22}$$

$$\boxed{C_v = 0.296}$$

$F_t = \sigma_{d_2} \times C_v \times b \times Y \times m$ (from eq 12.5 (a) Pg 204)

$4155.12 = 56 \times 0.296 \times 10m \times \pi\,(0.154 - 1.52 \times 10^{-3} m) \times m.$

$4155.12 = 520.75 m^2 \,(0.154 - 1.52 \times 10^{-3} m)$

$4155.12 = 80.19 m^2 - 0.791 m^3.$

$0.79 m^3 - 80.19 m^2 + 4155.12 = 0.$

$m = 100.86,\ 7.49,\ -6.96$

$\boxed{\text{module } m = 7.49}$

(from table 12.2 pg 229) standard module $\boxed{m = 8mm}$.

$b = 10m = 10 \times 8 \Rightarrow \boxed{b = 80mm}$

$m_1 = \dfrac{d_1}{Z_1} \Rightarrow 8 = \dfrac{120}{Z_1} \Rightarrow \boxed{Z_1 = 15 \text{ teeth}}$

$m_2 = \dfrac{d_2}{Z_2} \Rightarrow 8 = \dfrac{600}{Z_2} \Rightarrow \boxed{Z_2 = 75 \text{ teeth}}$

STEP : 4 :- check for dynamic

$F_d \quad F_t + \dfrac{K_3 V\,(Cb + F_t)}{K_3 V \sqrt{Cb + F_t}}$

$4155.12 + \dfrac{20.67 \times 7.22\,(303.52 \times 80 + 4155.12)}{20.67 \times 7.22 \sqrt{(303.52 \times 80 + 4155.12)}}$

(from table 12.13 pg 237)

class II $\quad e = 0.0386$

$0.03 = 235.9$

$0.0386 = C$

$(0.03 \times C) = (235.9 \times 0.0386)$

$\boxed{C = 303.52}$

$$\boxed{F_d = 17.52 \times 10^3 N}$$

STEP: 5 :- wear load.

$F_w \geq F_d$

$d_1 b Q K \geq F_d$

$(120 \times 80 \times 1.6 \times K) \geq 17.52.$

$\boxed{K = 1.14}$

$Q = \frac{2Z_2}{(Z_2+Z_1)}$

$Q = \frac{2 \times 75}{75+15}$

$\boxed{Q = 1.6}$

For cast steel and cast iron $\phi = 20°$ and $K \geq 1.14$.

For pinion CS BHN = 250
For gear CI BHN = 180.

7. A 24 teeth CS pinion rotating at 1500 rpm. drives an higher grade CI gear running at 500rpm. The teeth are 14.5° and module is 5mm face width is 36mm find the safe power that can be transmitted by this pair of spur gear.

Given data :-

P G.

$Z_1 = 24$

$N_1 = 1500$ rpm $N_2 = 500$rpm.

$b = 36$mm (high grade CI)

$\phi = 14.5°$

$m = 5$mm

$P = ?$

$Z_2 = ?$

$$\frac{N_1}{N_2} = \frac{Z_2}{Z_1} \Rightarrow \frac{1500}{500} = \frac{Z_2}{24} \Rightarrow \boxed{Z_2 = 72 \text{ teeths}}$$

$$d_1 = m \times Z_1 \Rightarrow 24 \times 5 \Rightarrow \boxed{d_1 = 120mm}$$

$$d_2 = m \times Z_2 \Rightarrow 5 \times 72 \Rightarrow \boxed{d_2 = 360mm}$$

Assume 0.20% of cast steel untreated

(from table 12.7 pg 234)

For pinion $\sigma d_1 = 138.3 \, MPa$

For gear $\sigma d_2 = 78.5 \, MPa$,

Lewis form factor $y = ?$

$$y_1 = 0.124 - \frac{0.624}{Z_1} \longrightarrow \text{(from 12.5 (e) Pg 204)}.$$

$$y_1 = 0.124 - \frac{0.624}{24}$$

$$\boxed{y_1 = 0.095} \quad \boxed{y_2 = 0.114}$$

STEP: 1 : Identify the weaker member.

Particle	σd	y	$\sigma d \times y$	remark
PINION	138.3	0.095	13.13	
GEAR	78.5	0.114	8.949	weaker.

Gear is the weaker member. The designed is based on gear.

STEP 2 :- Design is based on weaker member.

To find power (P)

WKT,

$$F_t = \frac{1000\,P\,C_s}{V}$$

According to lewis equation, the tangential tooth loa

$$V = \frac{\pi d_2 N_2}{60{,}000} = \frac{\pi \times 360 \times 500}{60{,}000} \Rightarrow \boxed{V = 9.42\ m/sec}$$

According to barth's formula velocity 13 m/s.

$$C_v = \frac{458}{458+V} = \frac{458}{458+942} \Rightarrow \boxed{C_v = 0.327}$$

$$Y = \pi y_2 = \pi \times (0.114) \Rightarrow \boxed{Y = 0.358}$$

$$F_t = \sigma_d\, C_v\, b\, Y\, m \longrightarrow (\text{eq } 12.5\ (a)\ pg\ 204)$$

$$= 78.5 \times 0.327 \times 36 \times 0.358 \times 5$$

$$\boxed{F_t = 1654.14\ N}$$

$$F_t = \frac{1000 \times P \times C_s}{V} = \frac{1000 \times P \times 1.5}{9.42} \Rightarrow \boxed{P = 10.38\ KW}$$

8. Design a pair of spur gear to transmit 24 KW @ 1000rpm to a parallel shaft to be rotated @ 400rpm The center distance between 2 shafts is 175mm.

Given data :-

$P = 24KW$

$N_1 = 1000 rpm$

$N_2 = 400 rpm$

$a = 175$

WKT,

$$i = \frac{N_1}{N_2} = \frac{1000}{400} \Rightarrow \boxed{i = 2.5}$$

$$i = \frac{d_2}{d_1} \Rightarrow 2.5 = \frac{d_2}{d_1} \Rightarrow \boxed{d_2 = 4d_1}$$

Centre Distance (a)

$$a = \frac{d_1 + d_2}{2} \Rightarrow 4 = \frac{d_1 + \cancel{4}^{2} d_1}{2} \Rightarrow 175 \times 2 = 5d \Rightarrow \boxed{d_1 = 70mm}$$

$$d_2 = 4 \times d_1 \Rightarrow d_2 = 4 \times 70 \Rightarrow \boxed{d_2 = 280mm}$$

Assume material for pinion and gear as CS 0.20% heat treated (from table 12.7 pg 234),

Allowable static stress for pinion $\boxed{\tau d_1 = 193.2 N/mm^2}$

Allowable static stress for gear $\boxed{\tau d_2 = 193.2 N/mm^2}$

Assume pressure angle $\phi = 20^\circ$ FDI system.

STEP 1 :- Identify the weaker member.

Since Both pinion and gear are made up of same material so. the design is based on pinion.

STEP 2 :- Design based on weaker member.

$$F_t = \sigma_d C_v b Y m \longrightarrow (eq^n\ 12.5(a)\ pg\ 204).$$

$$F_t = \frac{1000 \times P \times C_s}{V}$$

$$F_t = \frac{1000 \times 24 \times 1.5}{3.66}$$

$$\boxed{F_t = 9.83 \times 10^3 N}$$

$$V = \frac{\pi d_1 N_1}{60{,}000}$$

$$V = \frac{\pi \times 70 \times 1000}{60{,}000}$$

$$\boxed{V = 3.66\ m/sec}$$

$C_v = ?$

From barth's equation

$$C_v = \frac{3.05}{3.05 + V} \longrightarrow (eq\ 12.6\ (a)\ pg\ 205).$$

$$C_v = \frac{3.05}{3.05 + 3.66}$$

$$\boxed{C_v = 0.45}$$

$b = ?$

$b = 10m$

from lewis equation

$$y_1 = \left[0.154 - \frac{0.912}{Z_1}\right] \Rightarrow y_1 = \left[0.154 - \frac{0.912 \times m}{70}\right].$$

$y_1 = (0.154 - 0.01302 m)$

$F_t = \sigma_{d_1} C_v b Y m$

$= \sigma_{d_1} \times C_v \times b \times \pi y_1 \times m.$

$9.83 \times 10^3 = 193.2 \times 0.45 \times 10m \times \pi \times [0.154 - 0.01302 m) \times m.$

$9.83 \times 10^3 = 2.731 \times 10^3 m^2 (0.154 - 0.01302 m).$

$9.83 \times 10^3 = 420.57 m^2 - 35.54 m^3.$

$35.54 - 420.57 m^2 + 9.83 \times 10^3 = 0$

$m = -4.15, \ 7.99, \ 7.99.$

$\boxed{\text{module } m = 7.99}$

(from table 12.12 pg 229) standard module $\boxed{m = 8mm}$

$b = 10m = 10 \times 8 \Rightarrow \boxed{b = 80 \text{ mm}}$

$m_1 = \frac{d_1}{z_1} \Rightarrow 8 = \frac{70}{z_1} \Rightarrow \boxed{z_1 = 8.75 \text{ teeths}}$

$m_1 = \frac{d_2}{z_2} \Rightarrow 8 = \frac{280}{z_2} \Rightarrow \boxed{z_2 = 35 \text{ teeths}}$

Step 3: Dimensions:-

(from table 12.3 pg 229)

Addendum $(h_a) = m = \underline{8mm}.$

× deddendum $(h_f) = 1.25m = 1.25 \times 8 = \underline{10}$

× Tooth thickness $(t) = 1.5708m = 1.5708 \times 8 = \underline{12.56}$

dedendum $(h_f) = 1.2(8) = \underline{9.6}$

Tooth thickness $(t) = 1.5708 = \underline{12.56}$

working depth $= 2m = 2 \times 8 = \underline{16}$

Clearance $= 0.25 \times 8 = \underline{2}$

STEP 4 : Dynamic load (Fd) :-

$$Fd = Ft + F$$

$$Fd = Ft + \frac{K_3 v (Cb + Ft)}{K_3 v + \sqrt{Cb + Ft}} \longrightarrow \text{(eq 12.12 pg 209)}$$

$$9.83 \times 10^3 + \frac{20.67 \times 3.66 (441.71 \times 80 + 9.83 \times 10^3)}{20.67 \times 3.66 + \sqrt{441.71 \times 80 + 9.83 \times 10^3}}$$

$$\boxed{Fd = 21.68 \times 10^3 N}$$

(From table 12.13 pg 237

Class II $e = 0.0386$

$0.03 = 343.3$

$0.0386 = C$

$(0.03 \times C) = (343.3 \times 0.0386)$

$$\boxed{C = 441.71}$$

Step 5 :- Wear load.

$$Fw \geq Fd$$

$$d_1 b Q K \geq Fd$$

$$(70 \times 80 \times 1.6 \times K) \geq 21.68 \times 10^3$$

$$\boxed{K \geq 2.419}$$

$$Q = \frac{2Z_2}{(Z_2 + Z_1)}$$

$$Q = \frac{2 \times 35}{(35 + 8.75)}$$

$$\boxed{Q = 1.6}$$

For steel and steel $\phi = 20°$ FDI and $K \geq 2.419$
(from table 12.16 pg 239)

For pinion SS BHN =
For gear SS BHN =

9. A pair of spur gear with 20° FDI teeth consist of 20 teeth -Pinion meshing with 41 teeth gear the module is 3mm while b = 40mm, the material for pinion as well as gear is steel with ultimate tensile stress of 600 N/mm². the gears are heat treated to the surface hardness of 400 BHN The pinion rotates @ 140 rpm. and the service factor for the application is 1.75. Assume the velocity factor. accounting for dynamic load and the FOS is 1.5. determine the rated power that the gear can transmit.

Given data :-

$\phi = 20°$ FDI
$Z_1 = 20$ teeths
$Z_2 = 41$ teeths
$m = 3$mm
$b = 40$mm.
$\sigma_u = 600$ N/mm²
BHN = 400
$N_1 = 140$ rpm
$C_s = 1.75$
$C_v = 1.5$
FOS (n) = 1.5 $\quad$ P = ?

STEP : 1 :- Identify the weaker member.

since both pinion and gear are made up of same material. pinion is the weaker member. the designed is based on pinion, STEP 2 :- Design based on weaker member.

$$F_{t_1} = \tau_{d_1} \times C_v \times b \times Y_1 \times m \longrightarrow \text{(eq 12.5 (a) Pg 204)}$$

$$\tau_{d_1} = \frac{\tau_u}{FOS} = \frac{600}{1.5} \Rightarrow \boxed{\tau_{d_1} = 400\,N/mm^2}$$

$$y_1 = \left(0.154 - \frac{0.912}{Z_1}\right) \Rightarrow \left(0.154 - \frac{0.912}{20}\right) \Rightarrow \boxed{y_1 = 0.1084\,mm}$$

$$Y = \pi y_1 \Rightarrow \pi \times (0.1084) \Rightarrow \boxed{Y = 0.340}$$

$$F_{t_1} = 400 \times 1.5 \times 40 \times 0.340 \times 3$$

$$\boxed{F_{t_1} = 24480\,N}$$

$$V = \frac{\pi d_1 N_1}{60,000} \Rightarrow \frac{\pi \times (m Z_1) n_1}{60,000} \Rightarrow \frac{\pi \times (3 \times 20) 140}{60,000} \Rightarrow \boxed{V = 0.439\,m/s}$$

$$F_{t_1} = \frac{1000 \times P \times C_s}{V} \Rightarrow 24480 = \frac{1000 \times P \times 1.75}{0.439} \Rightarrow \boxed{P = 6.14\,KW}$$

10. Design a pair of steel spur gear required to transmit 12KW @ 2000rpm. of pinion, the velocity ratio required is 2.5:1 the allow static stress is for both the materials can be taken as 120MPa not less than 24 teeths are 20 stub tooth Involute System.

Given data :- (53)

$P = 12KW$

$N_1 = 200 rpm$

$i = 2.5 : 1$

$\sigma d_1 = 120 MPa$

$\sigma d_2 = 120 MPa$

$Z_1 = < 24$ teeths

$\phi = 20^\circ$ STI

Solution :-

$i = \frac{Z_2}{Z_1} \Rightarrow 2.5 = \frac{Z_2}{24} \Rightarrow \boxed{Z_2 = 60 \text{ teeths}}$

Lewis equation

$y_1 = 0.175 - \frac{0.95}{Z} \rightarrow$ (from eq 12.5(e) pg 204).

$= 0.175 - \frac{0.95}{25}$

$\boxed{y_1 = 0.135}$ $\boxed{y_2 = 0.159}$

STEP 1 : Identify the weaker member.

Since both pinion and gear are made up of same material. so the pinion is the weaker member. the design is based on pinion.

STEP 2 :- Design is based on weaker member.

Lewis eq^n for beam strength of tooth

$$m = \left[\frac{2mt}{\sigma d_1 \, Cv \, K \, Y_1 \, Z_1} \right]^{1/3} \longrightarrow \text{(eq 12.5 (b). pg 204)}.$$

mt = ?

$$mt = \frac{9.55 \times 10^6 \times P \times Cs}{n_1}$$

$$= \frac{9.55 \times 10^6 \times 12 \times 1.5}{2000}$$

$$\boxed{mt = 85.95 \times 10^3 \, N\text{-}mm}$$

$K = \frac{b}{m} = \frac{10m}{m} \Rightarrow \boxed{K = 10}$

$Y = \pi \times y_1 = \pi \times (0.135)$

$\boxed{Y = 0.424}$

$b = 10$

$$m = \left[\frac{2 \times 85.95 \times 10^3}{120 \times C_v \times 10 \times 0.42 \times 24} \right]^{1/3}$$

$$\boxed{m^3 C_v = 14.07}$$

$$m^3 = \frac{14.07}{0.5} \Rightarrow \boxed{m = 3.04 mm}$$

For spur gear

$C_v = 0.325$ to 0.6

$$\boxed{C_v = 0.5}$$

(From table 12.2 pg 229)

Assume preffered choice, standard module

$$\boxed{m = 4mm}$$

To check for module $(m) = ?$

$$d_1 = m \times Z_1 \Rightarrow 4 \times 24$$

$$\boxed{d_1 = 96mm}$$

$$V = \frac{\pi d_1 n_1}{60{,}000} \Rightarrow \frac{\pi \times 96 \times 2000}{60{,}000}$$

$$\boxed{V = 10.05 m/s}$$

$$C_v = \frac{4.58}{4.58 + V} = \frac{4.58}{4.58 + 10.05} \Rightarrow \boxed{C_v = 0.313}$$

$$(m^3 C_v)_{new} = (4^3 \times 0.313) \Rightarrow \boxed{(m^3 C_v)_{new} = 20.03}$$

Since $(m^3 C_v)_{new} > (m^3 C_v)_{req}$, design is safe.

STEP 3: Dimensions:-

Addendum $(h_a) = m = \underline{4mm}$

deddendum $(h_f) = 1.25m = 1.25 \times 4 = \underline{5}$

Tooth thickness $(t) = 1.5708m = 1.5708 \times 4 = \underline{6.28}$

Tooth space $= 1.5708m = 1.5708 \times 4 = \underline{6.28}$

working depth $= 2m = 2 \times 4 = \underline{8}$

whole depth $= 2.25m = 2.25 \times 4 = \underline{\underline{9}}$

clearance $= 0.25m = 0.25 \times 4 = \underline{\underline{1}}$

STEP4 :- Check for dynamic load $(Fd) = ?$

$$Fd = Ft_1 + \frac{K_3 v (Cb + Ft)}{K_3 v + \sqrt{Cb + Ft}} \rightarrow \text{(eq. 12.12 pg 209)}$$

$mt_1 = Ft_1 \times Y$

$mt_1 = Ft_1 \times d_1/2$

$85.95 \times 10^3 = Ft_1 \times \frac{96}{2}$

$\boxed{Ft_1 = 1790.62N}$

(from table 12.12 pg 237)

error $= 0.0254$

$0.02 = 228.9$

$0.0254 = C$

$\boxed{C = 290.70}$

$b = 10m = 10 \times 4 \Rightarrow \boxed{b = 40mm}$

$$Fd = 1790.62 + \frac{20.67 \times 10.05 (290.70 \times 40 + 1790.62)}{20.67 \times 10.05 + \sqrt{290.70 \times 40 + 1790.62}}$$

$\boxed{Fd = 10.40 \times 10^3 N}$

STEP5 :- wear load

$Fw \geq Fd$

$d_1 b Q K \geq Fd$

$96 \times 40 \times 1.42 \times K \geq 10.40 \times 10^3$

$\boxed{K = 1.907}$

$Q = \frac{2Z_2}{Z_1 + Z_2}$

$Q = \frac{2 \times 60}{60 + 24}$

$\boxed{Q = 1.42}$

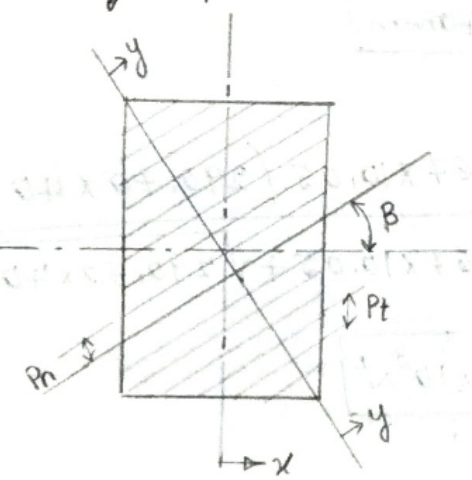
For steel and steel $\phi = 20^\circ$ FTI and $K \geq 1.907$.

(from table 12.16 pg 239)

For pinion steel BHN = 400.
For gear steel BHN = 350.

Helical Gear :-

helical gear are the gears in which teeth are cut in the form of helix around the gear, helical gears are used to connect parallel and non parallel shaft. helical gear picks up the load gradually resulting in smooth engagement and operation even @ high speed.



Terminologies :-

1. helix angle :- It is the angle b/w a element of the helix tooth and axis of rotation of the gear.
2. Transverse circular pitch (P) :- It is the distance b/w corresponding point on adjacent tooth measured on pitch circle (x-x (or) transverse plane).

3. **Normal circular pitch** :- It is distance b/w corresponding point on adjacent tooth measured in a distance normal to the tooth load (Y-Y normal plane).

i.e. normal pitch $\boxed{P_n = P\cos\beta}$ also. $\boxed{P_n = \pi m_n}$

4. **Diametral pitch** :- The diametral pitch is

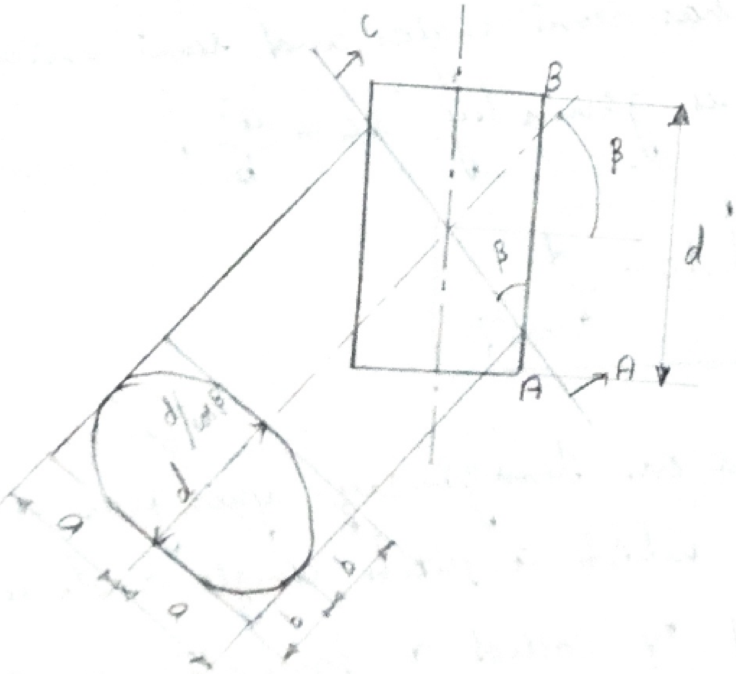
$$P_d = \frac{\frac{\pi}{P_n}}{\cos\beta} \Rightarrow \frac{\pi\cos\beta}{P_n} \Rightarrow P_d = \frac{\cancel{\pi}\cos\beta}{\cancel{\pi}m_n} \Rightarrow \boxed{P_d = \frac{\cos\beta}{m_n}}$$

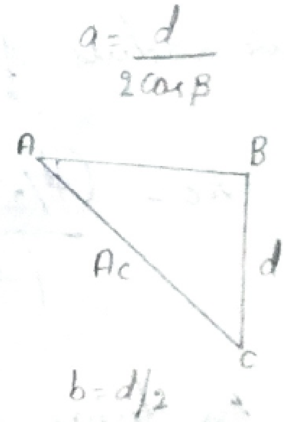
Also,

$$P_d = \frac{Z_1}{d_1} = \frac{Z_2}{d_2}$$

$$d_1 = \frac{Z_1}{d_2} = \frac{Z_1}{\frac{\cos\beta}{m_n}} = \frac{Z_1 m_n}{\cos\beta.} \Rightarrow \boxed{d_1 = \frac{Z_1 m_n}{\cos\beta}}$$

Virtual (or) Formative Number of teeths (or) Equivalent number of teeths (Z_e) :-





A plane parallel to the teeth i.e in normal plane the helical teeth will look like spur gear teeth In a normal plane the helix gear will be a spur gear having a definite number of teeths. The no of teeths (or) a equalent spur gear in a normal plane is known a virtual no of teeths.

The plane parallel to the axis of the gear is called as transvers plane of parallel to the tooth is called normal plane. The pitch plane of parallel to the tooth is called normal plane. The pitch cylinder of the helical gear is cut by a plane A-A which is normal to teeth element as shown in figure. The Intersecting plane A-A which is is normal to the tooth element of pitch cylinder produces an ellipse. The semi major and semi minor axis of the ellipse are $d/2\cos\beta$ and $d/2$ respectively.

For helix of a and b has semi major and semi minor axis radius of Carvature is given by $r_c = \frac{a^2}{b}$,

$$r_c = \frac{(d/2\cos\beta)^2}{(d/2)} \Rightarrow \boxed{r_c = \frac{d}{2\cos^2\beta}}$$

In design of helical gear an Imaginary spur gear is Considered in plane A-A which a pitch angle radius. (r_c) and module (m_n) it is called as formative @ virtual spur gear. The pitch circle angle dia is given by

$$d_c = 2r_c \Rightarrow 2 \times \frac{d}{2\cos^2\beta} \Rightarrow \boxed{d_c = \frac{d}{\cos^2\beta}}$$

The number of teeth (Z_e) on this imaginary spur gear is given by,

$$Z_e = \frac{2\pi r_c}{p_n} \Rightarrow Z_e = \frac{\cancel{2}\cancel{\pi}\frac{d}{\cancel{2}\cos^2\beta}}{\cancel{\pi} m_n} \Rightarrow Z_e = \frac{d}{m_n \cos^2\beta}$$

$$Z_e = \frac{\frac{Z m_n}{\cos\beta}}{m_n \cos^2\beta} \Rightarrow \boxed{Z_e = \frac{Z}{\cos^3\beta}}$$

where,

Z_e = virtual no of teeth on spur gear.

Z = actual no of teeth on helical gear.

β = helix angle.

Lewis form factor :-

Based on equivalent number of teeths.

$$y = \left(0.124 - \frac{0.684}{Z_e}\right) \longrightarrow \text{for } 14.5 \text{ involute system}$$

$$y = \left(0.154 - \frac{0.912}{Z_e}\right) \longrightarrow \text{for } 20^\circ \text{ involute system}$$

$$y = \left(0.175 - \frac{0.95}{Z_e}\right) \longrightarrow \text{for } 20^\circ \text{ stub tooth system}$$

Lewis eq^n for beam strength of helical gear:-

$$F_t = \frac{\sigma_d C_v b y}{P_{dn} C_w} \Rightarrow \frac{\sigma_d C_v b y m_n}{C_w} \Rightarrow \boxed{m_n = \frac{2 m_t C_w \cos\beta}{\sigma_d C_v K Y Z}}$$

According to Buckingham's the dynamic load on gear tooth. (from eq^n 12.26 (a) pg 214).

$$F_d = F_t + \frac{K_3 V (C b \cos^2\beta + F_t) \cos\beta}{K_3 V + \sqrt{C b \cos^2\beta + F_t}}$$

wear load F_w the limiting load for gear y is given by,

$$F_W = \frac{d_1 b Q K}{\cos^2\beta} \longrightarrow \text{(eq}^n \text{ 12.26 (c) pg 214).}$$

Problems:-

1. Design a pair of helical gear to transmit 12KW @ 1200rpm of pinion. the velocity ratio is 3:1 pinion has 24 teeths and it is made of 0.4 Carbon steel untreated. the gear is made of CS. the teeth are 14.5 Involute form in normal plane. helix angle is 25° ?

Given data:-

$P = 12KW$ material = 0.4% CS.

$n_1 = 1200rpm$ $\phi = 14.5°$

$n_2 = ?$ helix angle ($\beta = 25°$)

$i = 3:1$ $Z_1 = 24$.

Solution :-

$$i = \frac{N_1}{N_2} = 3 = \frac{1200}{N_2} \Rightarrow \boxed{n_2 = 400 rpm}$$

$$i = \frac{Z_2}{Z_1} = 3 = \frac{Z_2}{24} \Rightarrow \boxed{Z_2 = 72 \text{ teeths}}$$

(from table 12.22 Pg 241.) 0.4% cast steel untreated

$$\sigma_1 = 70 N/m^2 = 70 N/mm^2$$

$$\sigma_2 = 50 mN/m^2 = 50 N/mm^2$$

Virtual Number of teeths (Z_e)

$$Z_e = \frac{Z}{\cos^3 \beta} \longrightarrow \text{(eq 12.22 (a) Pg 211)}$$

$$Z_{e_1} = \frac{Z_1}{\cos^3 \beta} = \frac{24}{\cos^3 (25)} \Rightarrow \boxed{Z_{e_1} = 32.23}$$

$$Z_{e_2} = \frac{Z_2}{\cos^3 \beta} \Rightarrow \frac{72}{\cos^3 (25)} \Rightarrow \boxed{Z_{e_2} = 96.71}$$

Lewis form factor (y)

$$y = \left(0.124 - \frac{0.684}{Z_{e_1}}\right) \longrightarrow \text{(eq 12.5 (c) Pg 204)}$$

$$y_1 = \left(0.124 - \frac{0.684}{32.23}\right)$$

$$\boxed{y_1 = 0.102 mm}$$

$$y_2 = \left(0.124 - \frac{0.684}{Ze_2}\right) = \left(0.124 - \frac{0.684}{96.71}\right) \Rightarrow \boxed{y_2 = 0.116}$$

STEP 1 :- Identify the weaker member :-

Particular	σd	Y	$\sigma d \times Y$	remark.
P	70	0.102	7.14	
G	50	0.116	5.8 ✓	weaker.

Since the gear is the weaker member, so the designed is based on gear.

STEP 2 :- Design based on weaker member.

Lewis equation for beam strength of tooth.

$$m_n = \left[\frac{2M_t\, C_w \cos\beta}{\sigma d\, C_v\, K\, Y\, Z}\right]^{1/3} \longrightarrow (\text{eq } 12.24 \text{ (b) pg } 214)$$

$$Mt_2 = \frac{9.55 \times 10^6 \times 12 \times 1.5}{400} \Rightarrow \frac{9.55 \times 10^6 \times P \times C_s}{n_2} \Rightarrow \boxed{Mt_2 = 429.75 \times 10^3 \text{ N-mm}}$$

(from Table 12.21 pg 241)

$C_W = 1.15 \longrightarrow$ for Continuously lubricated,

$K = b/m_n \Rightarrow \boxed{K = 15}$

$b = 15m_n \longrightarrow$ for helical gear $(12.5\text{mm} \le m_n \le 20m_n)$ (12.23(f) pg 213)

$Y = \pi Y_2 = \pi \times (0.116) \Rightarrow \boxed{Y = \pi \times 0.116}$

$$M_n = \left[\frac{2 \times 429.75 \times 10^3 \times 1.15 \times \cos 25}{50 \times C_v \times 15 \times \pi \times 0.116 \times 72} \right]^{1/3}$$

$$\boxed{m_n^3 C_v = 45.57}$$

$(m_n^3 C_v) = m_n^3 (0.5) \Rightarrow 45.57 \Rightarrow \boxed{M_n = 45 mm}$

(from table 12.2 pg 229) std module $\boxed{M_n = 5 mm}$

$$d_2 = \frac{M_n Z_2}{\cos \beta} \longrightarrow (eq\ 12.19 (c)\ pg\ 211).$$

$$= \frac{5 \times 72}{\cos (25)} \Rightarrow \boxed{d_2 = 397.21 mm}$$

For velocity Between 5-10m/sec from birth's formula.

$$C_v = \frac{6.1}{6.1 + v} \longrightarrow (eq\ 12.25\ pg\ 214)$$

$$= \frac{6.1}{6.1 + 8.31}$$

$$\boxed{C_v = 0.423}$$

$$v = \frac{\pi d_2 n_2}{60{,}000}$$

$$= \frac{\pi \times 397.21 \times 400}{60{,}000}$$

$$\boxed{v = 8.31 m/s}$$

$(m_n^3 C_v)_{new} = (5^3 \times 0.423) \Rightarrow \boxed{(m_n^3 C_v)_{new} = 52.85}$

$$b_{min} = \frac{1.15 \pi M_n}{\sin \beta} \rightarrow ($$

$$= \frac{1.15 \times \pi \times 5}{\sin (25)} \Rightarrow \boxed{b_{min} = 42.74}$$

$b = 15 m_n = 15 \times 5 \Rightarrow \boxed{b = 75 mn}$

Since $b > b_{min}$ and $(m^3 C_v)_{new} > (m_n^3 C_w)_{reqr}$

STEP3 :- Dimensions :-

STEP4 :- Dynamic load (Fd) :-

$$F_d = F_t + \frac{K_3 V (C b \cos^2\beta + F_t)\cos\beta}{K_3 V + \sqrt{C b \cos^2\beta + F_t}} \rightarrow (\text{eq } 12.26 (a) \text{ pg } 214)$$

WKT,

$$F_{t_2} = \frac{\sigma_{d_2} C_v \times b \times Y \times m_n}{C_w} \longrightarrow (\text{eq } 12.24 (a) \text{ pg } 214)$$

$$= \frac{50 \times 423 \times 75 \times \pi \times 0.116 \times 5}{1.15}$$

$$\boxed{F_{t_2} = 2.513 \Rightarrow 2513.17 N}$$

(from table 12.12 pg 236) for s and s,

$e = 0.0277$,

$0.02 = 220.6$

$0.0277 = C$

$(0.02 \times C) = (0.0277 \times 220.6) \Rightarrow \boxed{C = 305.531}$

$$F_d = 2513.17 + \frac{20.67 \times 8.31 (305.531 \times 75 \times \cos(25)^2 + 2513.17) \times \cos(25)}{20.67 \times 8.31 + \sqrt{305.531 \times 75 \times \cos(25)^2 + 2513.17}}$$

$$\boxed{F_d = 12.95\,kN.}$$

STEP 5 :- Wear load (F_W):

$$F_W \geq F_d$$

$$\frac{d_1 b Q K}{\cos^2\beta} \geq 12.95 \times 10^3$$

$$\frac{132.405 \times 75 \times 1.5 \times K}{\cos^2(25)} \geq 12.95 \times 10^3$$

$$\boxed{K \geq 0.714}$$

$$d_1 = \frac{m_n Z_1}{\cos\beta}$$

$$= \frac{5 \times 24}{\cos(25)}$$

$$\boxed{d_1 = 132.405\,mm}$$

(from table 12.16 pg 239) S and S_1

For pinion steel and steel BHN = 300
For gear steel and steel BHN = 250.

2. Design a pair of helical Gear to transmit a power of 20KW. from a shaft running @ 1500rpm to a parallel shaft to run @ 450rpm. suggest suitable hardness for gear pair.

Solution

$P = 20KW$

$N_1 = 1500rpm$

$N_2 = 450rpm$

$$i = \frac{N_1}{N_2} = \frac{1500}{450} \Rightarrow \boxed{i = 3.33}$$

(From table 12.22 pg 241)

Assume material for both pinion & gear

Take 0.4 – 0.5% Carbon steel, untreated.

$$\boxed{\sigma_{d_1} = \sigma_{d_2} = 70N/mm^2}$$

Assume min number of teeth in pinion $\boxed{Z_1 = 20 \text{ teeth}}$

Assume helix angle $\boxed{\beta = 25^\circ} \rightarrow$ range $(20-45^\circ)$

Assume $\boxed{\phi = 20 FDI}$

$$i = \frac{Z_2}{Z_1} \Rightarrow 3.33 = \frac{Z_2}{20} \Rightarrow \boxed{Z_2 = 67 \text{ teeths}}$$

$$Z_{e_1} = \frac{Z_1}{\cos^3\beta} \Rightarrow \frac{20}{\cos^3(25)} \Rightarrow \boxed{Z_{e_1} = 26.86}$$

$$Z_{e_2} = \frac{Z_2}{\cos^3\beta} \Rightarrow \frac{67}{\cos^3(25)} \Rightarrow \boxed{Z_{e_2} = 90}$$

Lewis form factor (Y):

$$Y_1 = \left(0.154 - \frac{0.912}{Z_{e_1}}\right)$$

$$= \left(0.154 - \frac{0.912}{26.86}\right)$$

$$\boxed{Y_1 = 0.12 \text{ mm}}$$

$$Y_2 = \left(0.154 - \frac{0.912}{90}\right) \Rightarrow \boxed{Y_2 = 0.143 \text{mm}}$$

STEP 1 : Identify the weaker member.

Since both pinion and gear are made up of same material pinion is the weaker member.

STEP 2 :- Design is based on weaker member

$$M_n = \left[\frac{2M_t \cos\beta C_w}{\sigma_d \cdot C_v \cdot k \cdot Y \cdot z}\right]^{1/3} \longrightarrow (\text{eq } 12.24(b) \text{ Pg } 214)$$

$$M_t = \frac{9.55 \times 10^6 \times 20 \times 1.5}{1500} \Rightarrow \boxed{M_t = 191 \times 10^3 \ N\text{-}m}$$

(from table 12.21 pg 241)

$C_w = 1.15 \longrightarrow$ for Continuous lubrication

$k = b/m_n \Rightarrow \boxed{k = 15}$

$b = 20 m_n \longrightarrow$ for helical gear $(12.5mm \le m_n \le 20mm)$

$y = \pi y_2 = \pi \times (0.12) \Rightarrow \boxed{y = 0.376}$

$$M_n = \frac{2 \times 191 \times 10^3 \times 1.15 \times \cos 25}{70 \times C_v \times 15 \times 0.376 \times 67}$$

$$\boxed{m_n^3 C_v = 15.05}$$

$(m_n^3 C_v) = m_n^3 (0.5) \Rightarrow 15.05 \times (0.5) \Rightarrow \boxed{M_n = 3.11mm}$

(from table 12.2 pg 229) std module $\boxed{m_n = 5mm}$

$d_1 = \frac{m_n z_1}{\cos\beta} \longrightarrow$ (eq 12.19 (c) pg 211)

$= \frac{4 \times 20}{\cos(25)} \Rightarrow \boxed{d_1 = 88.27mm}$

For velocity between 5-10 m/s from barth's formula

$C_v = \frac{6.1}{6.1 + v} \longrightarrow$ (eq 12.25 pg 214)

$C_v = \frac{6.1}{6.1 + 6.93}$

$\boxed{C_v = 0.187}$

$$V = \frac{\pi d_1 n_1}{60{,}000} = \frac{\pi \times 88.27 \times 1500}{60{,}000} \Rightarrow \boxed{V = 6.932\, m/s}$$

$$(m^3 C_v)_{new} = 4^3 \times 0.187 \Rightarrow \boxed{(m^3 C_v)_{new} = 11.968}$$

Assume module $M = 5mm$.

$$(m_n^3 C_v)_{new} = 5^3 \times 0.187$$

$$\boxed{(m_n^3 C_v)_{new} = 23.375}$$

min face width of helix angle gear. according to AGMA.

$$b_{min} = \frac{1.15 \times \pi \times M_n}{\sin \beta} \rightarrow ($$

$$b_{min} = \frac{1.15 \pi \times 5}{\sin(25)} \Rightarrow \boxed{b_{min} = 42.74}$$

$$b = 15 M_n = 15 \times 5 \Rightarrow \boxed{b = 75 M_n}$$

Since $b > b_{min}$ and $(m^3 C_v)_{new} > (m_n^3 C_v)_{req.}$ so the design saf

STEP3: Dimensions:

STEP4: Dynamic load (F_d):

$$F_d = F_t + \frac{K_3 V (C b \cos^2\beta + F_t) \cos\beta}{K_3 V + \sqrt{C b \cos^2\beta + F_t}} \rightarrow (\text{eq } 12.26(a) \text{ pg } 214)$$

WKT, $$F_t = \frac{[\sigma_b] C_v \times b \times Y \times M_n}{C_w} \rightarrow (\text{eq } 12.24 (a) \text{ pg } 214)$$

$$= \frac{70 \times 0.187 \times 75 \times 0.376 \times 5}{1.15}$$

$$\boxed{F_t = 1604.94 N}$$

$$F_d = 160.94 + \frac{20.67 \times 6.93 \left(320.35 \times 75 \times \cos^2(25) + 160.94\right) \cos 25}{20.67 \times 6.93 + \sqrt{320.35 \times 75 \times \cos^2 25 + 160.94}}$$

$$\boxed{F_d = 10.18 \times 10^3 N}$$

STEP 5 :- wear load (F_W).

$F_W \geq F_d$.

$$\frac{130.33 \times 75 \times 0.45 \times K}{\cos^2(25)} = 10.18 \times 10^3$$

$$\boxed{K \geq 0.673}$$

BHN for pinion	=	250
BHN for gear.	=	200.

$$d_1 = \frac{M_n Z_1}{\cos\beta}$$

$$\frac{5 \times 20}{\cos(25)}$$

$$\boxed{d_1 = 110.33}$$

$C = 0.0277$

$0.02 = 237.3$

$0.0277 = C$

$$\boxed{C = 320.35}$$

3. A 24 teeth CS helical gear pinion drives a high grade CI. gear having 50 teeths. the teeths are 20° FDI. In a normal plane the helix angle is 45°, normal module is 3mm. find the safe power that can be transmitted by this gear @ a pinion speed of 500rpm.

Given data :-

$Z_1 = 24$

$Z_2 = 50$

$\phi = 20°$ FDI

$\beta = 45$

$M_n = 3mm.$

$P = ?$

solution :-

$$\frac{N_1}{N_2} = \frac{Z_2}{Z_1} \Rightarrow \frac{500}{N_2} = \frac{24}{50} \Rightarrow \boxed{N_2 = 240 rpm}$$

(from table 12,22 pg 241)

Allowable static stress $\boxed{\sigma_{d_1} = 50 MN/m^2}$ / mpa / N/mm^2

$$\boxed{\sigma_{d_2} = 30 N/m^2}$$

Lewis form Equation

$$y = \left(0.154 - \frac{0.912}{Z_{e_1}}\right) \longrightarrow \left(\text{eq } 12.5 \text{ (pg 204)}\right)$$

$$= \left(0.154 - \frac{0.912}{67.88}\right)$$

$$\boxed{y_1 = 0.140\text{mm}}$$

$$y_2 = \left(0.154 - \frac{0.912}{Z_{e_2}}\right)$$

$$= \left(0.154 - \frac{0.912}{141.42}\right)$$

$$\boxed{y_2 = 0.147\text{mm}}$$

(from eq 12.22 (a) pg 211)

$$Z_{e_1} = \frac{Z_1}{\cos^3\beta}$$

$$= \frac{24}{\cos^3(24)}$$

$$\boxed{Z_{e_1} = 67.84}$$

$$Z_{e_2} = \frac{50}{\cos^3(24)}$$

$$\boxed{Z_{e_2} = 141.42}$$

STEP 1: Identify the weaker member :-

Particular	σ_d	Y	$\sigma_d \times Y$	remark
P	50	0.140	7	
G	30	0.147	4.41	weaker

Since the gear is the weaker member. so the design is based on gear.

$$V = \frac{\pi d_2 N_2}{60.000} = \frac{\pi \times 212.13 \times 240}{60.000} \Rightarrow \boxed{V = 2.66\text{ m/s}}$$

$$d = \frac{Z_2 m_n}{\cos\beta} \longrightarrow \text{(eq 12.19 (c) pg 211)}$$

$$= \frac{50 \times 3}{\cos(45)}$$

$$\boxed{d = 212.13\,mm}$$

$$C_v = \frac{4.58}{4.58 + v} \longrightarrow \text{(eq 12.25 (a) pg 214)}$$

$$= \frac{4.58}{4.58 + 2.66} \Rightarrow \boxed{C_v = 0.632}$$

$b = 15 m_n$ – constant $\longrightarrow$ (range $12.5 m_n \leq m_n \leq 20 m_n$)

$$y = \pi y_2 = \pi \times 0.147 \Rightarrow \boxed{Y = 0.461}$$

$$F_t = \frac{\pi d \times C_v \times b \times Y \times M_n}{C_w} \longrightarrow (12.24(a)\ pg\ 214)$$

$C_w = 1.5$

Assume always continuous lubrication

$$= \frac{30 \times 0.632 \times 45 \times 0.461 \times 3}{1.15}$$

$$\boxed{F_t = 925.90}$$

$$F_t = \frac{1000 \times P \times C_s}{v} \longrightarrow \text{(from eq}^n \text{ 12.7 (a) pg 205)}$$

$$925.90 = \frac{1000 \times P \times 1.50}{2.66}$$

$$\boxed{P = 1.64\,KW}$$

TYPE 2.

3\. Design a pair of helical gear to transmit 30KW @ 300rpm of cast steel pinion. The transmission is 5 and the helix angle is 25°. The minimum pitch dia of any gear should be less than 125mm. The teeth are 14.5° Involute from the gear is made of high grade cast iron.

<u>Given data:</u>

$P = 30KW$.

$N_1 = 3000 rpm$.

$i = 5$.

$\phi = 14.5°$

$\beta = 25°$

$d_1 = 125 mm$.

Material for P = CS

for G = high grad CI

solution.

$i = \frac{N_1}{N_2} \Rightarrow 5 = \frac{3000}{N_2} \Rightarrow \boxed{N_2 = 600 rpm}$

$i = \frac{d_2}{d_1} \Rightarrow 5 = \frac{d_2}{125} \Rightarrow \boxed{d_2 = 625 mm}$

(from table 12.22 pg 241)

$\boxed{\tau d_1 = 50 N/mm^2}$

$\boxed{\tau d_2 = 30 N/mm^2}$

Assume temporarily no of teeth on pinion i.e $\boxed{Z_1 = 20 \text{ teeth}}$

$i = \frac{Z_2}{Z_1} \Rightarrow 5 = \frac{Z_2}{20} \Rightarrow \boxed{Z_2 = 100 \text{ teeth}}$

virtual no of teeth (Z_e)

$Z_e = \frac{Z_{e1}}{\cos^3\beta} = \frac{20}{\cos^3(25)} \Rightarrow \boxed{Z_{e1} = 26.86}$

$Z_e = \frac{Z_{e2}}{\cos^3\beta} = \frac{100}{\cos^3(25)} \Rightarrow \boxed{Z_{e2} = 134.32}$

Lewis form factor (y) :- for 14.5°

$$y = 0.124 - \frac{0.684}{Z_e} \longrightarrow$$

$$= 0.124 - \frac{0.684}{26.80}$$

$\boxed{y_1 = 0.098}$ $\boxed{y_2 = 0.118}$

STEP 1 : Identify the weaker member.

Particular	σ_d	y	$\sigma_d \times y$	remarks.
Pinion	50	0.098	4.9	
Gear	30	0.118	3.54	weaker.

Since the value of $\sigma_d \times Y$ of gear is less than pinion the design is based on gear.

STEP 2 :- Design is based on weaker member.

Lewis equation for helical gear.

$$F_t = \frac{\sigma_d \times C_v \times b \times Y \times m_n}{C_w} \longrightarrow \text{(eq 12.24 (a) pg 214)}$$

WKT,

$$F_t = \frac{1000 \times P \times C_s}{v}$$

$$= \frac{1000 \times 30 \times 1.5}{19.63}$$

$$\boxed{F_t = 2292.40\,N}$$

$$v = \frac{\pi d_2 n_2}{60,000}$$

$$= \frac{\pi \times 625 \times 600}{60,000}$$

$$\boxed{v = 19.63\,m/s}$$

From barth's formula for velocity B/w 10 to 20 m/s

$$C_v = \frac{15.25}{15.25+V} \longrightarrow (eq$$

$$= \frac{15.25}{15.25+19.63} \Rightarrow \boxed{C_v = 0.437}$$

$b = 15M_n \longrightarrow (12.5m_n \le M_n \le 20M_n)$.

$Y = \pi y_2$

$$y_2 = 0.124 - \frac{0.684}{Z_{e2}} \Rightarrow 0.124 - \frac{0.684}{\frac{d_2}{m_n \cos\beta}}$$

$$= 0.124 - \frac{0.684 \times M_n \times \cos^2\beta}{d_2}$$

$$y_2 = 0.124 - \frac{0.684 \times M_n \cos^2(25)}{625}$$

$$\boxed{y_2 = 0.124 - 8.989 \times 10^{-4} m_n}$$

$$Z_{e2} = \frac{Z}{\cos^3\beta}$$

WKT

$$Z_2 = \frac{d_2 \cos\beta}{M_n}$$

$$Z_{e2} = \frac{d_2 \cos\beta}{m_n \cos^3\beta}$$

$$\boxed{Z_{e2} = \frac{d_2}{M_n \cos^2\beta}}$$

$$2292.40 = \frac{30 \times 0.437 \times 15M_n \times \pi \times (0.124 - 8.98 \times 10^{-4} m_n) m_n}{1.15}$$

$$2636.26 = 617.79\,(0.124 - 8.98 \times 10^{-4} m_n^2)$$

$$2636.26 = 76.60m_n^2 - 0.554M_n^3$$

$$0.554m_n^3 - 76.60m^2 + 2636.26 = 0$$

$M_n = 138.01, \quad 5.99, \quad -5.74.$

(from table 12.2 pg 229) std module $\boxed{m_n = 6mm}$

$$Z_1 = \frac{d_1 \cos\beta}{m_n} = \frac{125 \times \cos 25}{6} \Rightarrow \boxed{Z_1 = 18.86 \simeq 19 \text{ teeth}}$$

$$Z_2 = \frac{d_2 \cos\beta}{m_n} = \frac{625 \times \cos 25}{6} \Rightarrow \boxed{Z_2 = 94.40 \simeq 94.40}$$

STEP 3 :- Dimensions:

STEP 4 :- Dynamic load (Fd)

$$F_d = F_t + \frac{K_3 V (C b \cos^2\beta + F_t)\cos\beta}{K_3 V + \sqrt{(C b \cos^2\beta + F_t)}} \rightarrow \text{(eq 12.26(a) pg 214)}$$

(from table 12.12 pg 236)

$e = 0.0316$

$0.03 = 227.4$

$0.0316 = C$

$(0.03 \times C) = (0.0316 \times 227.4)$

$$\boxed{C = 239.52}$$

$$F_d = 2292.40 + \frac{20.67 \times 19.63 (239.52 \times 90 \times \cos^2 25 + 229.40)\cos 25}{20.67 \times 19.63 + \sqrt{239.52 \times 90 \times \cos^2 25 + 2292.40}}$$

$\times \cos 25$

$$\boxed{F_d = 15.42 \times 10^3 N}$$

STEP 5 : wear load (Fw):

$$Q = \frac{2Z_2}{Z_1 + Z_2} = \frac{2 \times 95}{19 + 95} \Rightarrow \boxed{Q = 1.66}$$

$F_W \geq F_d$.

$$\frac{d_1 b Q K}{\cos^2\beta} \geq 15.42\times10^3$$

$$\frac{125.78\times90\times1.66\times K}{\cos^2(25)} \geq 15.42\times10^3$$

$$\boxed{K \geq 0.67.}$$

$$d_1 = \frac{m_n Z_1}{\cos\beta}$$

$$= \frac{6\times19}{\cos(25)}$$

$$\boxed{d_1 = 125.78}$$

$b = 15\times6 \Rightarrow \boxed{b = 90mm}$

(from table 12.16 pg 239)

BHN for pinion = 250
BHN for gear = 180

4. A pair of helical gear is to transmit 16KW the teeth are 20STI in diametral plane helix angle is 45°. pinion runs @ 10,000rpm., gear run @ 2500rpm. The centre distance is to be around 200mm the gears are made of CS. σ_d = 100MPa, σ_{en} = 618MPa. Determine module, face width. from static stress consideration and check for gear wear.

<u>Given data</u>

$P = 16KW$

$\phi = 20°$ STI

$\beta = 45°$

$N_1 = 10.000rpm$

$N_2 = 2500rpm$

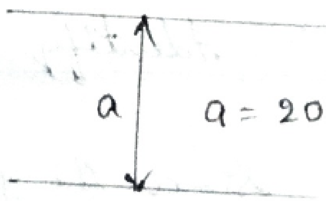
material = CS.

$a = 200MM$

$\sigma_{d_1} = 100MPa$

$\sigma_{d_2} = 100MPa$

$\sigma_{en} = 618MPa$.



$a = 200MM$.

Solution:-

$$i = \frac{N1}{N2} = \frac{10.000}{2500} \Rightarrow \boxed{i = 4}$$

For helical gear.

$$a = \left(\frac{Z_1 + Z_2}{2}\right) \cdot \frac{mn}{\cos\beta} \longrightarrow \left(eq\ 12.19\ (c)\ 211\right)$$

$$Z = \frac{d\cos\beta}{mn.}$$

$$= \frac{\left(\frac{d_1\cos\beta}{mn} + \frac{d_2\cos\beta}{mn}\right)}{2} \cdot \frac{mn.}{\cos\beta}$$

$$a = \frac{\cancel{\cos\beta}}{\cancel{mn}}\left(\frac{d_1 + d_2}{2}\right) \cdot \frac{\cancel{mn}}{\cancel{\cos\beta}}$$

$$\boxed{a = \frac{d_1 + d_2}{2.}}$$

WKT,

$$i = \frac{d_2}{d_1} = 4 * d_1 \Rightarrow d_2 = d_1 \times i.$$

$$a = \frac{d_2 + 4d_1}{2} \Rightarrow 200 = \frac{d_1 + 4d_1}{2} \Rightarrow \boxed{d_1 = 80mm.}$$

$$d_2 = 4 \times 80 \Rightarrow \boxed{d_2 = 320MM}$$

STEP 1 :- Identify the weaker member.

Since the gear and pinion are made up of same material the pinion is a weaker member.

STEP 2 :- Design is based on weaker member.

Lewis equation for helical gear.

$$F_t = \frac{\sigma_d \times C_v \times b \times Y \times m_n}{C_w} \longrightarrow \text{(eq 12.24 (a) pg 214)}$$

WKT.

$$F_t = \frac{1000 \times P \times C_s}{v}$$

$$= \frac{1000 \times 16 \times 1.5}{41.88}$$

$$\boxed{F_t = 573.06\,N}$$

$$v = \frac{\pi d_1 n_1}{60{,}000}$$

$$\frac{\pi \times 80 \times 1000}{60{,}000}$$

$$\boxed{v = 41.88\,m/s}$$

According to barth's formula ($v > 20\,m/s$)

$$C_v = \frac{5.55}{5.55 + \sqrt{v}} \longrightarrow \text{(eq 12.25 (d) pg 214)}$$

$$C_v = \frac{5.55}{5.55 + \sqrt{41.88}}$$

$$\boxed{C_v = 0.461}$$

Lewis Equation :-

$$y_1 = 0.175 - \frac{0.95}{Z_{e1}}$$

$$= 0.175 - \frac{0.95 \times m_n \times \cos^2(45)}{80}$$

$$\boxed{y_1 = 0.175 - 5.93 \times 10^{-3} m_n}$$

$$Z_{e1} = \frac{Z}{\cos^3\beta}$$

$$= \frac{d_1 \cos\beta}{m_n \cos^3\beta}$$

$$Z_{e1} = \frac{d_1}{m_n \cos^2\beta}.$$

$$Z_1 = \frac{d_1 \cos\beta}{m_n}$$

$$578.06 = \frac{100 \times 0.461 \times 15 m_n \times \pi \left(0.175 - 5.93 \times 10^{-3}\right) m_n}{1.15}$$

$$659.019 = 2172.41 m_n^2 \left(0.175 - 5.93 \times 10^{-3}\right) m_n$$

$$659.019 = 380.17 m_n^2 - 12.88 m_n^3$$

$$12.88 m_n^3 - 380.17 m_n^2 + 659.019$$

$$m_n = 29.457, \quad 1.3477, \quad -1.288$$

select least (+ve) value $\boxed{m_n = 1.34}$

(from table 12.2 pg 229) std module $\boxed{m = 1.5 mm}$

$$b = 15 m_n \Rightarrow 15 \times 1.5 \Rightarrow \boxed{b = 22.5 mm}$$

$$Z_1 = \frac{d_1 \cos\beta}{m_n} = \frac{80 \times \cos(45)}{1.5} \Rightarrow \boxed{Z_1 = 38 \text{ teeth}}$$

$$Z_2 = \frac{d_2 \cos\beta}{m_n} \Rightarrow \frac{320 \times \cos(45)}{1.5} \Rightarrow \boxed{Z_2 = 150.84 \text{ teeth}}$$

STEP 3 :- Dimensions:

STEP 4 : Dynamic load (Fd):

$$F_d = F_t + \frac{K_3 v \left(C_b \cdot \cos^2\beta + F_t\right) \cos\beta}{K_3 v + \sqrt{C_b \cdot \cos^2\beta + F_t'}} \rightarrow \text{(eq. 12.26 (a) pg 214)}$$

$$e = 0.0127$$

$$0.01 = 118.7$$

$$0.0127 = C$$

$$(0.01 \times C) = (0.0127 \times 118.7) \rightarrow \boxed{C = 150.74}$$

$$F_d = 573.06 + \frac{20.67 \times 41.88 \left(\frac{150.74 \times 22.5 \times \cos^2 45 + 573.06}{\cos 45}\right)}{20.67 \times 41.88 + \sqrt{150.74 \times 22.5 \times \cos^2 45 + 573.06}}$$

$$\boxed{F_d = 209.21 N}$$

STEP 5 :- Wear load (F_W)

$$F_W > F_d$$

$$Q = \frac{2Z_2}{Z_1 + Z_2}$$

$$= \frac{2 \times 38}{38 + 151}$$

$$\boxed{Q = 1.6}$$

$$K = \frac{\sigma_{cu}^2 \sin\alpha}{1.40} \left[\frac{1}{E_1} + \frac{1}{E_2}\right]$$

(eq 12.17 pg 214).

Assume youngs modulus of steel $E = 206.8 GPa$.

$$K = \frac{618^2 \times \sin(20)}{1.4} \left[\frac{1}{206.8 \times 10^3} + \frac{1}{206.8 \times 10^3}\right]$$

$$\boxed{K = 0.90}$$

$$F_W = \frac{d_1 b Q K}{\cos^2 \beta} = \frac{80 \times 22.5 \times 1.6 \times 0.90}{\cos^2(45)}$$

$$\boxed{F_W = 5184 \geq F_W.}$$

Since $F_W \geq F_d$ the design is safe.

4. A pair of helical gear have 20° pressure angle in the normal plane, the normal module is 5mm and the module in diametrical plane is 5.77mm. The pitch diameter of smallest gear is 115.47mm. If the transmission ratio is 4:1.

Determine,,,

helix angle of normal pitch, transfer pitch number of teeth on each gear, addendum, deddendum, whole depth center distance, base circle diameter, clearance, outside diameter, working depth, root circle diameter.